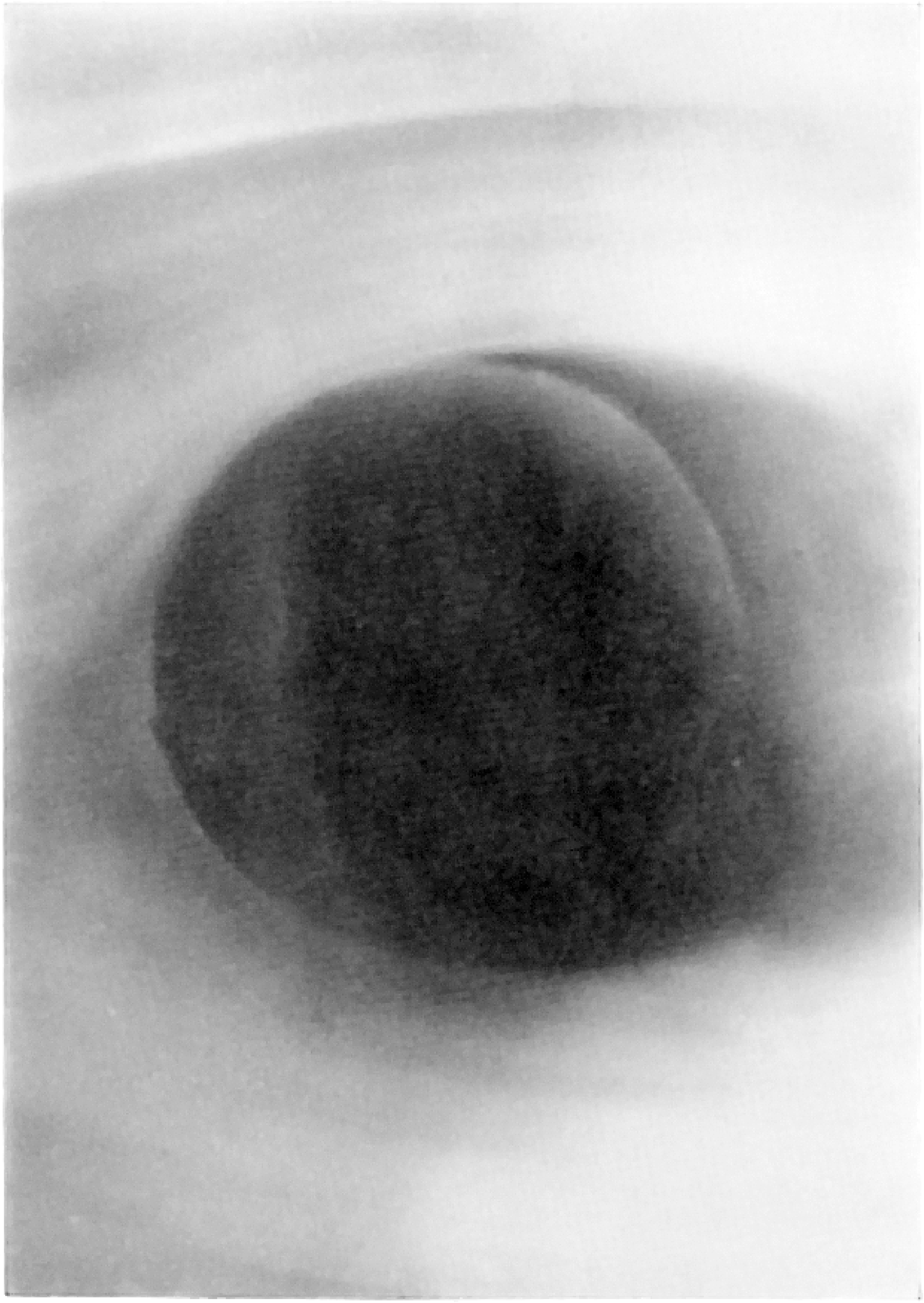
AERODYNAMICS

BY

F. W. LANCASTER



PHOTOGRAPH SHOWING FLOW OF AIR ROUND A CYLINDER IN MOTION.

[Frontispiece.

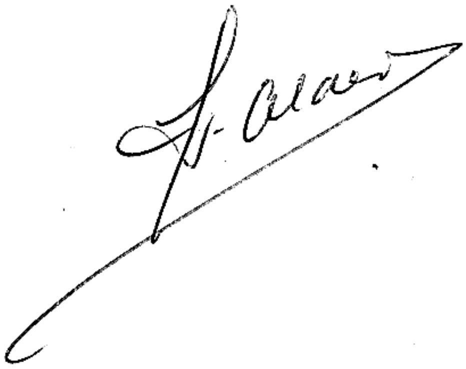
AERODYNAMICS

CONSTITUTING THE FIRST VOLUME OF A COMPLETE WORK ON AERIAL FLIGHT

BY

F. W. LANCASTER

With Appendices on the Velocity and Momentum of Sound Waves, on the Theory of Soaring Flight, etc.



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PREFACE.

The problems that arise in connection with the study of Aerial Flight are so numerous and of so diverse a character that, except for their relation to the title subject, they would scarcely find place in one volume. In the present work an attempt is made, it is believed for the first time, to treat the classification of the phenomena associated with the study of Flight on a comprehensive and scientific basis,

The origin of the present work may be said to date from some experiments carried out in the sear 1894. These experiments, which were primarily directed as a test of certain theoretical views which the author then advanced, resulted in the production of flying models of remarkable stability, whose equilibrium was not destroyed by an ordinary gale of wind,

As originally formulated the theory was incomplete and in many ways imperfect, but it has been developed from time to time during the last twelve years to an extent that to-day renders the approximately correct proportioning of an *aerodrome*[1] a matter of straightforward calculation.

The author has found the question of publication one of some difficulty. At first it was intended to arrange and pi1blish the investigations simply in order of date, theoretical work being accompanied so far as possible by appropriate experimental

[1] A word derived from the Greek, αερο–δρομοσ (lit. *"traversing the air"* or *"an air – runner"*), proposed by the late Prof. Langley to denote a gliding appliance or flying machine; hence also *aerodromics*, the science specifically involved in the problems connected with *free flight*. The word aerodrome has been grossly misapplied by Continental writers to denote a balloon shed. The author considers that from its derivation the word *aerodromics* may be given a more comprehensive meaning than that originally proposed, perhaps even to include both the *aerodynamics* and *aerodonetics* of flight. The question is merely one of terminology. (Compare Glossary, p. 393.)

demonstration. It soon became evident that there were considerable *lacunae*, and these were filled by subsequent investigations, the scope of the work being greatly extended. Finally it was decided to make the publication a complete treatise on *Aerial Flight*, the main classification being as follows:—

Vol. I. *Aerodynamics*, relating to the theory of aerodynamic support and the resistance of bodies in motion in a fluid.

Vol. II. *Aerodonetics*[1] or *Aerodromics*, dealing with the forms of natura1 flight path, with the questions of equilibrium and stability in flight, and with the phenomenon of "soaring."

So far as has been found possible the work has been modelled on *non-mathematical lines*. The commonly distinctive feature of a modern *mathematical* treatise, in any branch of physics, is that the investigation of any problem is initially conducted on the widest and most comprehensive basis, equations being first obtained in their most general form, the simpler and more obvious cases being allowed to follow naturally, the greater including the less. The reader who is only moderately equipped with mathematical knowledge is thus frequently at a loss to comprehend the initial stages of the argument, and so has no great chance of fully appreciating the conclusions.

It is impossible, in connection with the present subject, to avoid the frequent use of mathematical reasoning, and occasionally the non-mathematical reader may find himself out of his depth. The author has endeavoured to minimise any difficulty on this score by dealing initially with the simpler cases and afterwards working up to the more general solutions; and further by the careful statement of all propositions apart from mathematical expression, and by the re-statement of conclusions in non-mathematical language. Wherever appropriate, numerical examples are given in order to move completely elucidate the methods employed and the results attained.[2]

1 Derived from the Greek, αεροδοντοσ (lit. ''tossed in mid-air," "soaring").

2 A passage occurs in the preface to Poynting and Thomson's "Sound" that may be quoted as being to the point:—

"Even for the reader who is mathematically trained, there is some advantage in the study of elementary methods compensating for their cumbrous

Whenever the author has consciously derived assistance from the work of previous investigators, due acknowledgment has been made ; the present work is, however, in the main, a connected series of personal investigations. Should the author inadvertently have put forward as new, results that have been previously published or methods that have been previously employed, he can at least claim in mitigation of the offence that very many of the present investigations were actually done more than ten years ago; the work has only been withheld to the present date in order that publication might take the form of a complete and connected account of the mechanical principles of flight such as could be the Letter understood by, and be of the greater service to, the Scientific and Engineering World.

In offering to the public the first instalment of the present work, the author desires to record his conviction that the time is near when the study of Aerial Flight will take its place as one of the foremost of the applied sciences, one of which the underlying principles furnish some of the nest beautiful and fascinating problems in the whole domain of practical dynamics.

In order that real and consistent progress should be made in Aerodynamics and Aerodonetics, apart from their application in the engineering problem of mechanical Bight, it is desirab1e, if not essential, that provision should be made for the special and systematic study of those subjects in one or more of our great Universities, provision in the form of an adequate endowment with proper scope for its employment under an effective and enlightened administration.

The importance of this matter entitles it to rank almost as a National obligation; for the country in which facilities are given for the proper theoretical and experimental study of flight will inevitably find itself in the best position to take the lead in its application and practical development. That this must be

form. They bring before ns more evidently the points at which the various assumptions are made, and they render more prominent the conditions under which the theory holds good."

considered a. vital question from a National point of view is beyond dispute; under the conditions of the near future the command of the air must become at least as essential to the safety of the Empire as will be our continued supremacy on the high seas.

The present volume deals exclusively with the Aerodynamics of Flight ; the arrangement of this section is as follows:—

Chapters I., II. and III, are devoted to the preliminary exposition of the underlying principles of fluid dynamics, examined from different points of view. Chapter I. is of an introductory character, and includes a discussion of to the nature of *fluid resistance*, the theory of the Newtonian medium, and a preliminary examination of the questions of *discontinuous motion* and *stream-line form*. Chapter II. is devoted to the consideration of *viscosity* and *skin-friction*, the argument being largely founded on dimensional theory; and Chapter III. consists in the main of an account of the *Eulerian hydrodynamic theory*, in which the mathematical demonstrations are in general taken for granted;[1] this chapter also includes some further discussion of the phenomenon of *discontinuous flow* and a review of the controversy relating to same.

Chapter IV. consists in most part of an investigation on *peripteral motion*,[2] dating from the year 1894—5 and offered to the Physical Society of London in the year 1897, but rejected.[3]

[1] The reader is referred to "Hydrodynamics" (Horace Lamb, Cambridge University Press) for the complete mathematical treatment: a work to which the author desires to acknowledge his indebtedness.

[2] A term proposed and employed by the author to denote the type of fluid motion generated in the vicinity of a bird's wing, or the supporting member of an aerodrome essential to its supporting function (lit. ronnd about the wing," Gr, peri and pteron). The term has an architectural signification which can by no possibility clash with its present usage.

[3] The rejection of this paper was probably due to an unfortunate selection of the readers to whom it was submitted. The names of the Society's readers are not disclosed, but from the wording of the reports (which the author is not at liberty to quote), it would scorn that the recognised application of the Newtonian method (as in the Theory of propulsion) was a thing unknown to them.

The hydrodynamic interpretation included in the present work has been added subsequently, and the latter portion of the original paper has been revised and rewritten on the more secure basis thus afforded.

Chapters V. and VI. constitute a *résumé* of that which is known concerning the *aeroplane* treated both from a theoretical and experimental standpoint.

Chapters VII. and VIII. present, for the first time, a series of investigations made by the author (dating from 1894, 1898, and 1902, but not previously published) of the principles governing the *economics of flight*, and their application in the correct proportioning of the supporting member; these investigations are based on the peripteral theory of Chapter IV. aided by a hypothesis, being in the main an adaptation of Newtonian method.[1]

Chapter IX. includes, with a discussion on the elementary theory of propulsion, an origina1 investigation on the theory of the *screw propeller* founded on the peripteral theory of Chapters IV., VII., and VIII; This theory leads to results that are in remarkable accord with experience, and enables a useful series of rules to be laid down as a guide to design; applied to the marine propeller, the theory gives a form quite in harmony with modern practice. The chapter concludes with a dissertation on the subject of the *expenditure of power in flight*.

Charter X., with which the present volume concludes, is of the character of an appendix, being an account of the more important of the experimental researches in aerodynamics published to date, and to which references have been made in the body of the work. This chapter also includes an account of some hitherto

[1] The essentially Newtonian character of all methods based on the principle of the direct communication of momentum, in hydrodynamics, is not so widely recognised as it ought to be. Thus the Rankine-Froude theory of propulsion is a simple and legitimate application of the Newtonian theory (see Chap. IX.). Newton was careful to specify the nature of the medium essential to the rigid app1ication of his method. (prop. xxxiv., Book II., *Enunciation*); subsequent writers have unfortunately not been so careful , and. error has resulted.

unpublished experiments by the author, and some criticism of the conclusions formulated by earlier investigators.

A few terminological innovations have been made at one time and another, as necessity has arisen. New words, or words bearing a special or restricted meaning, are given in the glossary following Chapter X., in addition to the usual footnote references.

Numerical work has been done by the aid of an ordinary 25 c.m. slide rule, with a liability to error of about $^1/_5$th of 1 per cent., an amount which is quite unimportant.

The author desires to express his thanks to Mr. P. L. Gray in connection with the preparation of the present volume for the Press, in particular for his most welcome assistance in the examination and correction of the proof sheets.

BIRMINGHAM,
October 1907.

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AERODYNAMICS.

CHAPTER I.

FLUID RESISTANCE AND ITS ASSOCIATED PHENOMENA.

§ 1. Introductory.—A body in motion through a fluid of any kind, whether liquid or gaseous, experiences resistance, and work has to be done in its propulsion.

Such resistance is due to two clearly distinct causes, the independent nature of which may be illustrated by considering a few commonplace instances.

If a piece of cardboard be moved briskly through the air, the resistance, though quite sensible, is very much less than that experienced when a similar movement is attempted under water. In this case the difference is evidently due to the very much greater density of water, which at 10° C. is 800 times that of air. If now we similarly compare water with any ordinary grade of heavy lubricating oil, or with common treacle, we again find a great difference, but this time the density is approximately the same, and we recognise that the resistance is due to an. entirely different cause a certain *stickiness* of the medium, otherwise *viscosity.* All fluids are viscous to a greater or lesser degree; the viscosity of water is small, that of air still legs, whilst lubricating oil and treacle are highly viscous substances.

Now just as in the study of ordinary *mechanics* it is found expedient initially to neglect the effects of friction, so, in connection with the present subject, we can afford to ignore the *fluid friction* to which the viscosity-of the fluid gives rise, and in the first instance deal with resistance as a function of density alone.

The analogy here suggested is not complete. It frequently happens in the ease of fluids that the effects of viscosity have to be taken into account as part of the general dynamic system; consequently it is sometimes necessary to devote some attention to these effects even in the preliminary discussion.

The question of compressibility is one on which also it is desirable to have some convention. It is popularly supposed that whereas gases are *compressible*, liquids are virtually *incompressible*; no broad distinction of this kind is justified. The criterion of compressibility in fluid dynamics involves the relative density of the fluid, and on this basis air is only *about* eighteen times as compressible as water, the ratio of the vo1ocity of sound in water and air being approximately in the proportion of $\sqrt{18}:1$. It is shown later that the influence of compressibility only becomes manifest as the velocity of motion approaches the velocity of sound in the fluid in question, or if the pressures developed involve a serious change in density.

The velocities and pressures ordinarily involved in aerial flight are such as will justify the initial assumption that the air Is incompressible that is to say, that the influence of the compressibility is negligible . The possibility at error resulting from this hypothesis will be considered subsequently. (Appendix I.)

§ 2. Two Methods.—There are two ways in which problems in fluid dynamics may be approached: (1) By the method of the *Newtonian medium* ; this, though of great service in certain special cases, is not strictly applicable to real fluids. (2) By the methods of Euler and Lagrange, by which complete equations of motion are obtained, defining the flow of the fluid in the three co-ordinate dimensions of space. This is the method employed in works on analytical hydrodynamics, and discussed in Chap. III. of the present work.

The basis o[the Newtonian method is found in the principle of the conservation of momentum, which may be taken as corollary to the third law of motion as written: When force acts on a body the momentum generated in unit time is proportional to the force. This method is best studied in connection with a hypothetical medium suggested by Newton, on which he based several of the problems in the "Principia." This medium is defined as consisting of a large number of material particles, equally distributed in space, having no sensible magnitude, but possessing mass; the particles are not supposed to act upon or be connected to each other in any way. Bodies traversing a region filled with this medium experience a resistance which is proportional to the momentum communicated per second, and is a quantity that can be calculated mathematically, provided that the velocity of the body and the density of the medium be known, and the surface in presentation of the body be defined.

The employment of this method and its deficiencies in the case of a real fluid are illustrated in the case of the *normal plane* (Chap. V.), where it is found to give a considerably greater pressure value than actually obtains; the general form of the pressure law is, however, in approximate accord.

§ 3. The Newtonian Method.—Employing absolute units, let F = the resistance, let m = mass acted upon during time t, and v the velocity in the line of motion imparted to the mass m; then the fundamental equation is: $F = \frac{mv}{t}$.

Now so long as we are dealing with a simple body of mass m, and imparting to it a velocity v, the above equation is merely a statement of the law of motion cited, any constant being eliminated by the fact that we are employing absolute units. The equation, however, holds good whatever the number of parts into which the mass be divided, and however the velocities of the different parts vary amongst themselves. In this case the expression may be written: $F = \frac{\sum(mv)}{t}$. The proof is as follows:—Let us suppose that the mass acted on per second be divided into n parts, and that each part be acted on by the force F for $1/n$ th of a second. Then the momentum communicated to each part = F/n, and the total momentum per second = $n\frac{F}{n} = F$, which holds good when the number of parts becomes infinite and the communication of momentum continuous. And since the communication of momentum for each of the periods of 1/n th second is independent of the masses of the individual parts, it is in nowise essential that the *n* parts are of equal mass; consequently the velocities acquired by the different parts may vary amongst themselves to any extent, without thereby affecting the total quantity of momentum communicated.

This principle in its application to fluid dynamics has sometimes been termed the *Doctrine of the Continuous Communication of Momentum*.

§ 4. Application of the Newtonian Method in the Case of′ the Normal p1ane.—To illustrate the method in the case of the normal plane in motion in a region supposed filled with the medium of Newton, we must first define the mode in which the surface of plane the imparts velocity to the constituent particles.[1] If, on the one hand, the body and the particles be supposed perfectly elastic, then the particles on colliding with the surface will bounce off with a velocity equal and opposite (relatively) to that with which they strike; that is to say, if V be the velocity of the plane, and v be the velocity given to the projected particles, v will be double of V. If, on the other band, we suppose that the plane is inelastic, and that it eats up or absorbs the particles on impact, then the velocity imparted to them will be equal to that of the plane, or, $v = V$. It is thus of little consequence which hypothesis we take, the one will give a result exactly twice as great as the other. We will select the second hypothesis, which will give the lesser value of the two.

[1] Compare "Principia," prop. xxxv., Book II.

Let us assume the medium as of the same density as air at 14° C., and 760 m.m. pressure, that is to say, let one cubic foot contain $\frac{1}{13}$ th lb. mass. Let P represent the pressure per square foot, that is the total force F divided by the area of the plane. Then the mass dealt with per second to develop a force P will be $\frac{V}{13}$ and the velocity v being equal to V we have:—

Momentum per second = $P = \frac{V^2}{13}$.

But P here is in absolute units, *poundals*. Reducing to pounds, we have:—

Pressure = $\frac{V^2}{13g} = \frac{V^2}{13 \times 32.2} = \frac{V^2}{420}$ approximately. If the velocity be expressed in *miles per hour*, this becomes $\frac{V^2}{200}$ (nearly). This may be recognised at once as a result often given in text books as the pressure-velocity equation for air, and is tacitly put forward as if founded on experiment. It is approximately 50 per cent. higher than the true value.

If, instead of introducing a value for the density, we denote this by ρ, the expression (absolute units) is: $P = \rho V^2$; the experimental value is, in the case of air, $P = .7\rho V^2$, or, in the case of water, $P = .55\rho V^2$, as ascertained for flat plates of compact outline, (See Chap. V.)

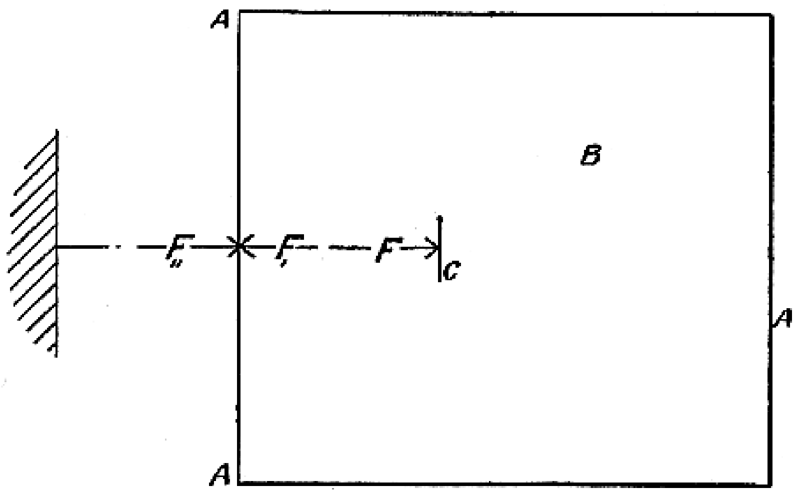
§ 5. Deficiency of the Newtonian Method.—It is evident from the -foregoing that the theory of the Newtonian medium is capable of giving results within measure of the truth, when applied to real fluids. The degree of accuracy varies with the circumstances, and the author will now endeavour to point out the reasons why, and the manner in which, the method fails, and indicate the circumstances under which the Newtonian theory is applicable and those under which it is not.

At the outset it may be set down that any defect in the theory is due, not to any want of exactitude in the fundamental theory—this rests definitely on the third law of motion and is absolute—but rather to the difficulty and uncertainty as to its manner of application in the case of real fluids.

The nature of this difficulty is clearly demon3trated by the following proposition:—

When a body propelled through an incompressible fluid, contained within a fixed enclosure, experiences resistance to its motion, the force exerted by the body on the fluid does not impart momentum to the fluid, but is transmitted instantly to the confines of the fluid ho ever remote, and is wholly borne by its boundary surfaces.

Let us suppose (Fig. 1) a body which we will take to be a normal plane *C*, acted upon by a force *F* in an enclosure *A*,



filled with fluid *B*. The enclosure may be supposed as large as we please, or, in the limit, infinite in its dimensions.

Then the condition that the enclosure is fixed denotes that the force *F* applied to the plane is applied *from* the walls of the enclosure; for, if we suppose it applied *from without* we can resolve the force into a force acting between the plane and the enclosure $F\ F_I$ and a force of equal magnitude acting *from without* on the enclosure F_{II}, and since the enclosure is fixed the latter can have no effect.

Now since the fluid is incompressible its density is constant and uniform, therefore the mass centre of the contents cannot move relatively to the enclosure and the enclosure itself is fixed, consequently the fluid in sum does not receive momentum.

If, in place of a real fluid possessing continuity, we had supposed the enclosure filled with the medium of Newton, then momentum would have been communicated in sum to the particles of the medium, and the resistance could be calculated in the manner already demonstrated.

If we take away the condition that the enclosure is *fixed* and suppose the force applied *from without*, then the problem is not essentially altered, for though the external force *F* will now impart momentum to the system *en bloc*, its action in this respect has no relation to phenomena in the interior, and does not provide any data for the determination of the pressure-velocity relation.

The supposition that the body is a *plane* evades any question relative to the density of the body itself, and thus simplifies the argument. This question could also be eliminated by supposing the body to possess the same density as the surrounding fluid; in any case a force applied to the body to overcome its inertia is a matter external to, and without influence on, the conditions.

The foregoing proposition cannot depend in any way upon the viscosity or otherwise of the fluid; the existence of viscosity can affect the mode of transmission of the force and the velocity of the body that accompanies its transmission, but can have no influence on the total force transmitted.

It is thus apparent that no momentum is imparted to an actual fluid in the sense that it is imparted to the Newtonian medium, and this is the real cause of the difficulty in the application of the Newtonian method.

The principle here demonstrated is referred to in the present work as the "Principle of No Momentum."

§ 6. Illustrations of the Principle of No Momentum.—The foregoing proposition is of moment in connection with several problems in fluid dynamics, and presents the subject in an aspect that is somewhat unfamiliar. Its import may be pointed by the following illustrations.

A body of apparent weight *F*, falls uniformly through a column of *inviscid* or frictionless fluid, contained in a vertical cylindrical or prismatic vessel, open at the top. Then the weight of the body (*F*) will be carried as additional pressure on the base of the vessel during the whole time of the descent; and if the vessel be tall and narrow the additional pressure will be approximate1y uniform and equivalent to an additional "head"; if it be wide, so that the walls are remote from the body, then the distribution on the pressure area will not be uniform, but will be greatest at the point vertically beneath the body, and less at points more remote. If the fluid possess viscosity, the whole of the force *F* may not reach the base of the vessel, but will in part be borne by its walls, but the total force carried by walls and base will in any ease be equal to *F*.

In the above illustration there is nothing that is strikingly unfamiliar If we suppose the vessel to be an ordinary jar of liquid, placed in the scale pan of a balance, there is a certain obviousness in the fact stated; the weight of the whole will be just the same whether the weight rests inert at the base of the jar, or whether it be falling uniformly through the fluid. When, however, the principle is applied to bodies aerodynamically supported in the free atmosphere the matter is not so self-evident; here, for example, we find that the weight of a parachutist is borne by the earth's surface almost from the moment he leaves the car, and his presence overhead, or the presence of a passing flight of birds, could be detected barometrically if we possessed an instrument of sufficient delicacy.

§ 7. Transmission of Force.—Comparison of Fluid and Solid.— We know that we may look upon a solid in stress as communicating momentum since it transmits force, but a distinction must be drawn. When the flow of momentum is equal and opposite, as in the case of a solid in stress, there is no displacement. of matter, and it is only when there is a displacement of matter that the Newtonian method can be applied. The case of a gas under pressure is, according to the kinetic theory, an example of the actual communication of momentum, and its pressure and the mean velocity can be correlated on the Newtonian principle but once lose sight of the transference of matter (molecular motion), and we can only assert that the gas is exerting and transmitting force.

As a whole, the fluid, in the previous section, does not gain or lose momentum any more than does a cast-iron pillar supporting a load. The stress is transmitted in part by viscosity and in part dynamically; the part that is transmitted dynamically is transmitted by an actual transference of momentum from certain parts of the fluid to certain other parts; but this we cannot follow without equating the motions of the fluid throughout the whole of the enclosed space. The manner in which a portion of the stress is transmitted by viscosity may be compared, if we adopt a view put forward by Poisson and Maxwell, to its transference by a solid continually giving way in shear; or, on the other band, if the fluid is gaseous, we may, on the kinetic theory, regard the viscous resistance as of purely dynamic origin, but belonging to a system quite apart from that of the aerodynamic disturbance.

§ 8. When the Newtonian Method is Applicable.—In the case of the Newtonian medium the quantity of matter dealt with, and momentum imparted per unit time, are defined quantities; but in the real fluid it has been shown that the motion produced is a circulation of the fluid not accompanied by any total change of momentum, and although parts of the fluid receive momentum in the direction of the applied force, other parts receive momentum in the opposite direction. In spite of this difficulty, there are certain cases in which the principle of the continuous communication of momentum can be applied. A most striking example is to be found in the theory of marine propulsion founded by Rankine and Froude.

According to this theory the propeller (whether screw, paddle, or jet propulsion be employed) is taken as operating an a certain mass of fluid per second, to which it imparts a certain sternward velocity. It is assumed that the momentum per second so imparted constitutes and accounts for the whole propulsive force, an assumption that under practical conditions is doubtless very close to the truth. In the case of the screw propeller the mass of fluid per second is calculated from the volume of the cylindrical body of water defined by the track of an imaginary circle drawn through the tips of the blades; in other forms of propulsion similar approximate methods of assessment are adopted.

The sternward velocity imparted to the fluid by the propeller is, under proper conditions, small in comparison with the velocity of travel, so that the lines of flow are not radically altered, and instead of a circulation such as arises in the case of a normal plane, there is merely a slight contraction of the stream at the region in which the propeller operates, and a trifling readjustment of the surrounding lines of flow to suit.

In general it would appear that the Newtonian method is applicable in cases where the volume of the fluid handled is great, but where the impressed velocity is small in comparison with the velocity of motion, and where there are well-defined conditions on which to compute the amount of fluid dealt with per second, it is found to be entirely deficient in dealing with the resistance of bodies of smooth contour, or "streamline form," such as may now be discussed.

§ 9. On Streamline Form.—When a body of fish-shaped or ichtyoid form travels in the direction of its axis through a frictionless fluid there is no disturbance left in its wake. Now we have seen that in any case the fluid as a whole receives no momentum, so that it is perhaps scarcely legitimate to argue that there is no resistance because there is no communication of momentum, although this is a common statement.[1] It is clear, however, that if there is no residuary

[1] This somewhat academic objection would cease to apply if any means could be found to properly define the idea which undoubtedly is conveyed to the mind by the argument in question.

disturbance there is no necessary expenditure of energy, aild this equally implies that the resistance is *nil*.

The fluid in the vicinity of a streamline body is of necessity in a state of motion and contains energy, but this energy is conserved, and accompanies the body in its travels, just as in the case of the energy of a wave. It adds to the kinetic energy of the body in motion just as would an addition to its mass.

According to the mathematical theory of Euler and Lagrange, all bodies are of streamline form. This conclusion, which would otherwise constitute a *reductio ad absurdum*, is usually explained on the ground that the fluid of theory is inviscid, whereas real fluids

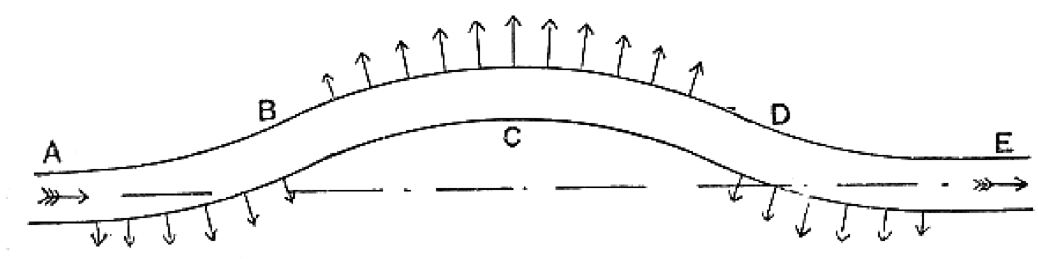


FIG. 2.

possess viscosity. It is questionable whether this explanation alone is adequate.

§ 10. Froude's Demonstration.—An explanation of the manner of the conservation of kinetic energy, in the case of a streamline body, has been given by the late Mr. W. Froude.

Referring to Fig. 2, *A*, *B*, *C*, *D*, *E*, represents a bent pipe, through which a. fluid is supposed to flow, say in the direction of the lettering, the direction at *A* and at *E* being in the same straight line; it is assumed that the fluid is frictionless. Now so long as the bends in the pipe are sufficiently gradual, we know that they cause no sensible resistance to the motion of the fluid. We have excluded viscous resistance by hypothesis, and if the areas at the points *A* and *E* are equal there is no change in the kinetic energy. Moreover, the sectional area of the pipe between the points *A* and *E* may vary so long as the variations are gradual; change of pressure will accompany change of area on well-known hydrodynamic principles, but no net resistance is introduced; consequently the motion of the fluid through the pipe does not involve any energy expenditure whatever.

Let as now examine the forces exerted by the fluid on different portions of the pipe in its passage. The path of the particles of fluid in the length between the points A and B is such as denotes upward acceleration, and consequently the fluid here must be acted on by an upward force supplied by the walls of the pipe, and the reaction exerted by the fluid on the pipe is equal and opposite. A shorter way is to regard this reaction as the centrifugal component of the curvilinear path of the flow, and as such it may be indicated by arrows as in the figure.

By assuming the bends in the pipe to be equal and a uniform velocity throughout, it follows that those centrifugal components exactly balance one another, each to each, and the pipe has no unbalanced force tending to push it in one direction or the other. The argument may be found presented in this form in White's "Naval Architecture." The same net result follows, no matter what the exact form of the bends, or whether or no the velocity is uniform, provided the bends are smooth and the cross-section (and therefore the velocity) is the same at *E* as at *A*, for under these circumstances the pressure at *A* will he the same as at *E*, the applied forces thus being balanced, and there will be no momentum communicated by the fluid in its passage.

When a streamline body travels through a fluid the lines of flow may be regarded as passing round it as if conveyed by a number of pipes as in Fig. 2. It is convenient, and it in nowise alters the problem, to look upon the body as stationary in an infinite stream of fluid (Fig. 3); we are than able to show clearly the lines of flow relatively to the surface of the body. Now let us take first the fluid stream that skirts the surface itself, and let us suppose this included between the walls of an imaginary pipe, then forces will be developed in a manner represented in Fig. 2, and these forces may be taken as acting on the surface of the body. It is not necessary to suppose that there is actual tension in the fluid, as might be imagined from Fig. 3, where the forces act outward from body, this is obviated by the general hydrostatic pressure that obtains in the region; the forces as drawn are those supplied by the motion of the fluid, and can be looked upon as superposed on those due to the static pressure.

If, similarly, we deal with the next surrounding layer of fluid, we find that the pressure to which it gives rise acts to reinforce that of the layer underneath (i.e., nearer the body), and so on, just as in hydrostatics the pressure is continually increased by

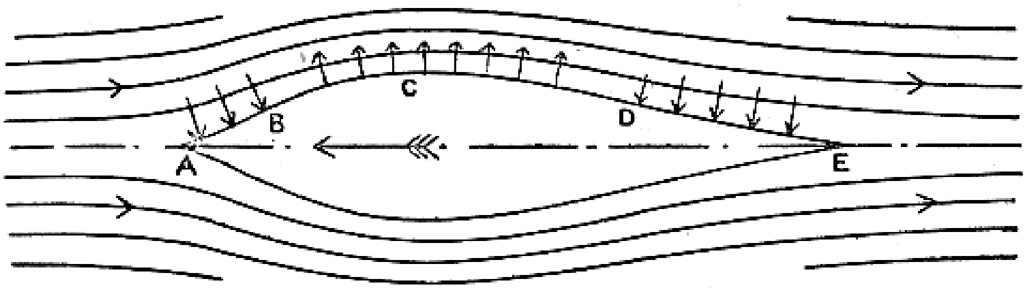


FIG. 3.

the addition of superincumbent layers of fluid, and thus we find that the body is subjected to increased pressure acting on its front and rear, and diminished pressure over its middle portion. Now it has been shown, in the case of the pipe, that the algebraic sum of all forces in the line of notion is zero, so that in the streamline body the sum of the forces produced by the pressure on its surfaces will be zero, that is to say, it will experience no resistance in its motion through the fluid.

It may be taken as corollary to the above, that in a viscous fluid the resistance of a body of streamline form will be represented approximately by the tangential resistance of its exposed area as determined for a flat plate of the same general proportions. This is the form of allowance suggested by Froude; a more elaborate and accurate method has been given by Rankine, in which allowance is made for the variation in the velocity of the fluid at different points on the surface of the body. Neither of these methods includes any allowance for viscous loss owing to the distortion of the fluid in the vicinity of the body.

§ 11. The Transference of Energy by the Body.—It is of interest to examine the question of the transference of energy through the streamline body itself from one part of the fluid to the other. For the purpose of reference the different portions of the body have been named as in Fig. 4, the *head*, the *shoulder*, the *buttock*, and the *tail*, the head and shoulder together being termed (as in naval architecture) the *entrance*, and the buttock and tail the *run*. The dividing line between the entrance and run is situated at the point of maximum section, and the dividing line between the

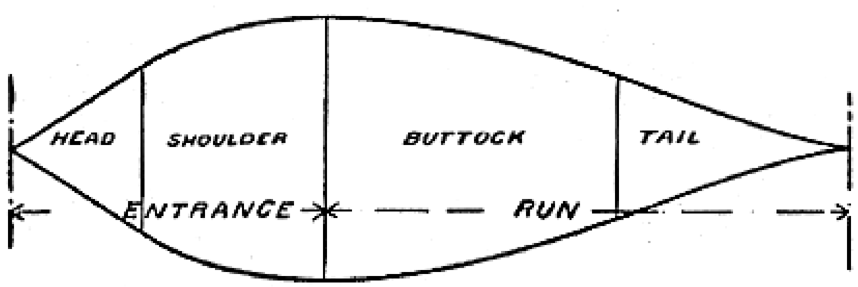


FIG. 4.

head and shoulder on the one hand, and between the buttock and tail on the other, is the line on this surface of the body at which the pressure is that of the hydrostatic "head."

Now, as the body advances, the head, being subject to pressure in excess of that due to the hydrostatic "head," is therefore doing work on the fluid that is to say, transmitting energy to the fluid; the shoulder also advancing towards the fluid is subject to pressure less than that due to hydrostatic head, and is consequently receiving energy from the fluid; the buttock, which is receding from the fluid, is also a region of minus pressure and so does work on the fluid; and lastly, the tail is receding under excess pressure and so receives energy. We thus see that there are two regions, the head and buttock, that give up energy continuously to the fluid, and two regions, the shoulder and tail, that continuously receive it back again. The condition of perfect streamline motion is that the energy account shall balance.

§ 12. Need for Hydrostatic Pressure.—Cavitation.—The motion impressed on the fluid by the pressure region of the head is compulsory, unless (as may happen in the case of a navigable balloon) deformation of the envelope can take place. The motion impressed by the shoulder, on the contrary, depends upon hydrostatic pressure, for otherwise there is no obligation on the part of the fluid to follow the surface of the body hydrostatic pressure is necessary to prevent the formation of a void. The pressure measured from the real zero must everywhere be positive, otherwise the fluid will become *discontinuous* and cease to follow the surface. This is a difficulty that has been actually experienced in connection with screw propellers, and termed *cavitation*.

§ 13. The Motion in the Fluid.—It has been shown that the head of a streamline form is surrounded by a region of increased pressure. Consequently the fluid as it approaches this region will have its velocity reduced, and the streamlines will widen out, as shown in Fig. B (see also Figs. 42, 44, 45, etc.). This behaviour of the fluid illustrates a point of considerable importance, which is frequently overlooked. Whenever a body is moving in a fluid its influence becomes sensible considerably in advance of the position it happens to occupy at any instant. The particles of fluid, commence to adjust themselves to the impending change with just as much certainty as if this body acted directly on the distant particles through some independent agency, and when the body itself arrives on the scene the motion of the fluid is already conformable to its surfaces. There is no *impact*, as is the case with the Newtonian medium, and the pressure distribution is more often than not quite different from what might be predicted on the Newtonian basis. This behaviour of a fluid is due to its continuity.

It follows from elementary considerations that the fluid in the "amidships" region possesses a velocity greater than the general velocity of the fluid (the body, as before, being reckoned stationary). We know that at and about the region C, Fig. 3, the fluid has a less area through which to pass than at other

points in the field of flow. It is in sum less than the normal area. of the stream by the area of cross-section of the body at the point chosen. But the field of flow is made up of a vast number of tubes of flow, so that in general each tube of flow will be contracted to a greater or less extent, the area of section of the tubes being less at points where the area of the body section is greater. We know that a contraction in a tube of flow denotes an increase of velocity.

Thus on the whole the velocity of the fluid is augmented across any *normal* plane that intersects the body itself, but the increase of velocity is not in any sense uniform in its distribution. In fact towards the extremities of the body, and in its immediate neighbourhood, we have already seen that the motion of the fluid is actually *slower* than the general stream,

The motion of the fluid is examined from a quantitative point of view in a subsequent chapter (Chap. III.), where plottings are given of the hydrodynamic solution in certain cases.

§ 14. A Question of Relative Motion.—The motion of the fluid has so far been considered from the point of view of an observer fixed relatively to the body; it will be found instructive to examine the same motions from the standpoint of the fluid itself, that is to say, to treat the problem literally as a *body moving through a fluid*, instead of as a *fluid in motion* round a fixed body.

It is evident that the difference is merely one of relative motion. The problems are identical: we require to consider the motions as plotted on co-ordinates belonging to the fluid instead of co-ordinates fixed to the body itself. The relation of the *streamlines* (which we have so far discussed) to the paths of motion (which we now propose to examine) is analogous to that of the cycloid or trochoid to its generating circle.

§ 15. Displacement of the Fluid.—An unfamiliar effect of the passage of a body through a fluid is a *permanent displacement* of the fluid particles. This displacement may he readily demonstrated. If a mass of fluid be moved from any one part of an enclosure to any other part, the enclosure being supposed filled with fluid, there is a circulation of fluid from one side to the other during transit ; and if we suppose it to be moved from one side to the other of an imaginary barrier surface, then an equal volume of fluid must cross the same barrier surface in the opposite direction. Now it is of no importance whether the thing we move be a volume of fluid or a solid body, so that when a streamline body passes from one side to the other of a surface composed of adjacent particles of fluid, that surface will undergo displacement in the reverse direction to that in which the body is moving, and the volume included between the positions occupied before and after transit will be equal to the volume of the body itself.

Moreover, since the actual transference of the fluid is due to a circulation from the advancing to the receding side of the body, it will take place principally in the immediate vicinity of the body and less in regions more remote; it is, therefore, immaterial whether the fluid be contained within an enclosure or whether one or more of its confines be free surfaces, provided that continuity is maintained, and that the body is not in the vicinity of a free surface.

§ 16. Orbital Motion of the Fluid Particles.—Since the motion of the fluid results in a permanent displacement, the motion of a particle does not, strictly speaking, constitute an orbit, It is, however, convenient in cases such as the present to speak of the motion as orbital.

If could follow the path of a particle along any streamline, and note its change of position relatively to an imaginary particle moving in the path and with the velocity of the undisturbed stream, we should have data for plotting the orbital motion corresponding to the particular streamline chosen. Thus we know that the amplitude of the orbit of any particle, measured at right angles to the direction of flight, is equal to that of the corresponding streamline.

We further know that, in general, the particles have a retrograde motion—that is, their final position is astern of their initial position—also that the maximum retrograde velocity is to be found in the region of maximum amplitude. Beyond this we know that the initial motion of any particle is in the same direction as that of the body, and that this initial motion is greater for particles near the axis of flight than for those far away.

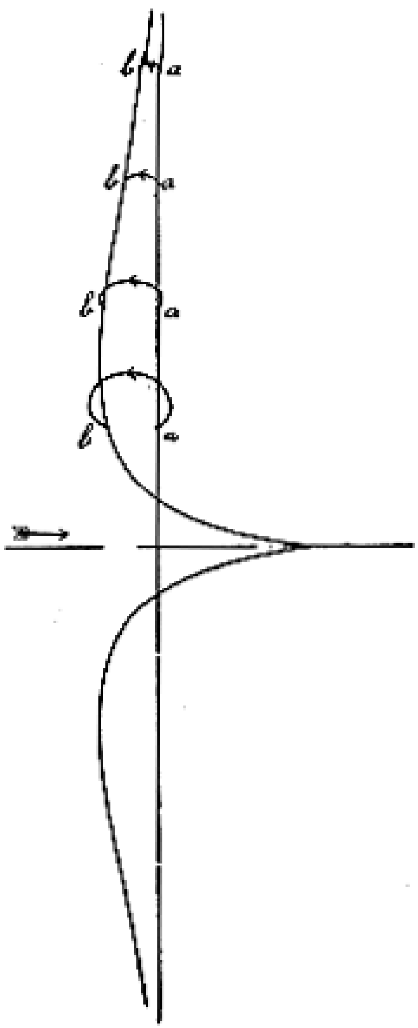


FIG. 5.

Let *b, b, b,* etc., Fig. 5, represent the final position of a series of particles originally situated in the plane *a, a, a*; then the orbits of these particles will originate on the plane *a, a, a,* and terminate on the surface *b, b, b,* and the motion will be of the character shown.

The form of the surface *b, b, b,* will be different for different forms of body. It will evidently approach the plane *a, a, a,* asymptotically, and generally will tend to form a cusp pointing along the axis of flight. The development of this cusp is greatest in cases where the extreme entrance and run are of bluff form, as in the Rankine Oval, Fig. 42, where the point of the cusp is never reached, the surface approaching the axis of flight asymptotically. In reckoning the displacement of the fluid (§ 15), the volume included in the cusped surface forward of the plane a, a, must be considered negative, since here the fluid is displaced in the same direction as the motion of the body.

§ 17. Orbital Motion and Displacement,—Experimental Demonstration.—The displacement of the fluid and the form of the orbit can be roughly demonstrated by a simple smoke experiment. If a smoke cloud be viewed against a dark background during the passage of a body of streamline form in its vicinity, the retrograde movement of the air is clearly visible. So long as the surface of the body is not too close, the movement is clean and precise, and the general character of the orbit form can be clearly made out; it is found to be, so far as the eye can judge, in complete accord with the foregoing theory. The commencement and end of the orbit, where the motion should be in the same direction as the body, is most difficult to observe, though even this detail is visible if the orbit selected be sufficiently near to the axis off flight. The difficulty here is that the latter part of the orbit is generally lost in consequence of the "frictional wake,"[1] *i.e.*, the current set up by viscous stress in the immediate neighbourhood of the body in motion. In all actual fluids a wake current of this kind is set up, and the displacement surface b, b, b, Fig. 5, is obliterated in the neighbourhood of its cusp by a region of turbulence.

§ 18. Orbital Motion,—Rankine's Investigation.—The form of the orbits of the fluid particles has been investigated theoretically for a certain class of body by Rankine (Phil. Trans., 1864).

Rankine closely studied the streamlines of a body of oval form, generated by a certain method from two foci (§ 77) and by calculation arrived at the equation to the orbit motion of the particles. The result gives a curve whose general appearance is given in Fig. 6 (actual plotting), in which the arrows represent the motion of the particle, the direction of motion of the body being from left to right.

Discussing the particular case in which the eccentricity of the oval vanishes, and the form merges into that of a circle, Rankine says,—" . . . The curvature of the orbit varies as the distance of the

[1] A term used in naval architecture.

particle from a line parallel to the axis of *X*, and midway between that axis and the undisturbed position of the particle. This is the property of the looped or coiled elastic curve; therefore when the water-lines are cyclogenous the orbit of each particle of water forms one loop of an elastic curve." Further, he says—"The particle starts from *a*, is at first pushed forward, then deviates outwards and turns backwards, moving directly against the motion of the solid body as it passes the point of greatest breadth as shown. The particle then turns inwards, and ends by following the body, coming to rest at *b* in advance of its original position."

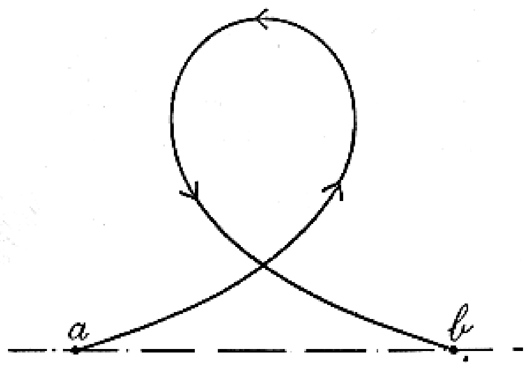


FIG. 6.

This orbit in some respects resembles that arrived at by the author, but differs in the one very important point that, whereas the author's method gives a *retrograde* displacement of the fluid as the net consequence of the passage of the body, Rankine's conclusion is exactly the contrary.

As the author's result is capable of experimental verification, it is evident that some subtle error must exist in Rankine's argument, the exact nature of which it is difficult to ascertain.

§ 19. Bodies of Imperfect Streamline Form.—In an actual fluid, bodies of other than streamline form experience resistance apart from that directly due to viscosity.

In the practical shaping of a streamline body it is found essential to avoid corners or sharp curves in the line of flow. Bodies in which due precaution is not taken in this respect offer considerable resistance to motion, and the regions of abrupt curvature give rise to a discontinuity in the motion of the fluid. Thus Fig. 7 represents a double cone moving axially, and it will be noticed that the flow has not time to close in round the run, as it would do in a properly formed streamline body, but shoots past the sharp edge, indicated in the figure. The region in the rear of the body, *Z*, is filled with fluid that does not partake of the general flow, and which is termed *dead-water*.

The resistance experienced by bodies imperfect form is due to the work done fluid, which is not subsequently given back, as is the case with the streamline body. This resistance can he traced to two causes, namely, excess pressure on the surface in presentation and diminished pressure in the dead-water region. The former is of dynamic origin, the energy being expended in directly impressing motion on portions of the fluid; the latter is due to the *entrainment* or viscous drag experienced by the dead - -water at the surface bounded by the live stream. It is generally believed that, in a fluid whose viscosity is negligible, the latter cause would be inoperative, the whole resistance being then due to tine excess pressure region in front of the body, the dead-water or wake being at approximately the hydrostatic pressure of the fluid.

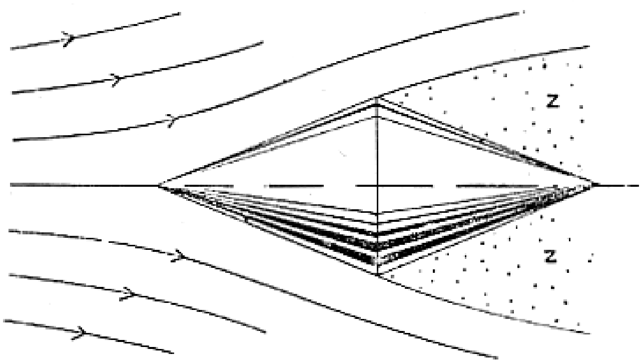


FIG. 7.

The surface separating the live stream and the dead-water constitutes *discontinuity* since the *velocity* of the fluid, considered as a function of its position in space, is discontinuous.

This case is not one of a *physical discontinuity*, such as discussed in § 12, for the region on either side of the surface is filled with the same kind of fluid; it is rather a *kinetic discontinuity*, that is to say a *discontinuity of motion*.

§ 20. The Doctrine of Kinetic Discontinuity.—The theory at kinetic is of modern origin, having been introduced and developed by Kirchhoff, Helmholtz, and others, to account for the *phenomenon* *of resistance* in fluid motion. The analytical theory, based on the hypothesis of continuity, does not in general lead to results in harmony with experience. All bodies, according to the Eulerian theory, are of streamline form, provided that the hydrostatic pressure of the fluid is sufficient to prevent cavitation; we know that in practice this is not the case.

According to the teaching of Helmholtz and Kirchhoff, a kinetic discontinuity can be treated as if it were a physical discontinuity ; that is to say, the contents of the dead-water region can be ignored; and this method of treatment is now generally recognised, although not universally so. The controversial aspect of the subject is discussed at length at the conclusion of Chap. III.

The principal objection to the theory of discontinuity is that in any inviscid fluid a surface of discontinuity involves rotation, and therefore, by a certain theorem of Lagrange, it is a condition that cannot he generated.[1] A further objection sometimes raised is that such a condition as that contemplated would be unstable, and that the surface of discontinuity, even if formed, would break up into a multitude of eddies. Whether this is the case or not in an inviscid fluid, it is certain that in a fluid possessed of viscosity a surface of discontinuity does commence to break up from the instant of its formation; but as this breaking up does not affect the problem iii any important degree, the objection in the case of the inviscid fluid is probably also without weight.

In a real fluid a finite difference of velocity on opposite sides of any surface would betoken an infinite tangential force. Con—sequent1y the discontinuity becomes a stratum rather than a surface, and the stratum will either be a region in which a velocity gradient exists (§ 31), or it will become the seat of turbulent motion (§ 37), the latter in all probability.

The conception of the discontinuity as a surface and the method involving this conception are in no way affected by these considerations. The term surface if discontinuity may be looked

[1] Chap. III. §§65-71.

upon as an abstraction of that which is essential in a somewhat complex phenomenon.

§ 21. Experimental Demonstration of Kinetic Discontinuity.— The reality and importance of the discontinuous type of motion can be demonstrated conclusively by experiment.

In Fig. 8, *a, b, c,* is a hollow spherical globe in which *d* is a tube arranged to project in the manner shown. An ordinary lamp globe and chimney will be found to answer the purpose the former having

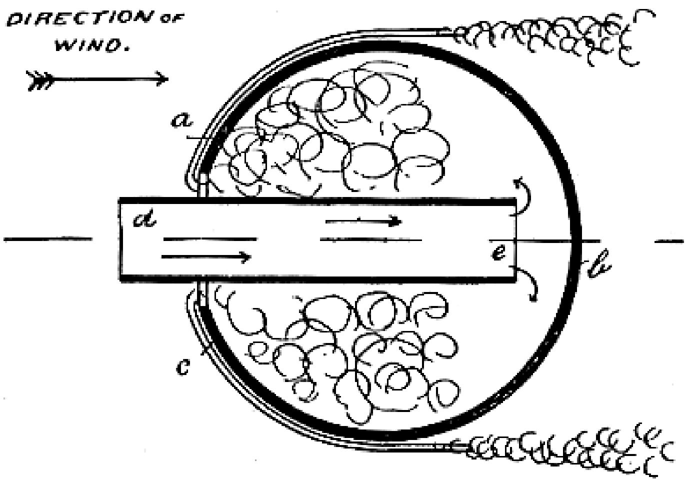


FIG. 8.

one of its apertures closed by a paper disc. The whole is carefully filled with smoke and then moved through the air in a direction from right to left, the relative direction of the air being indicated by the arrow.

It will be found that the air will enter the tube and displace the smoke through the annular aperture The issuing smoke follows the surface of the sphere in the most approved manner as far as the "equator," but then passes away at a tangent, the stratum of discontinuity, the dead-water region, and the turbulent character of motion, being all clearly manifest. The discontinuity, as may have been anticipated, does not appear as a clean-cut surface; it is marked almost from the commencement, as indicated in the figure, by eddy

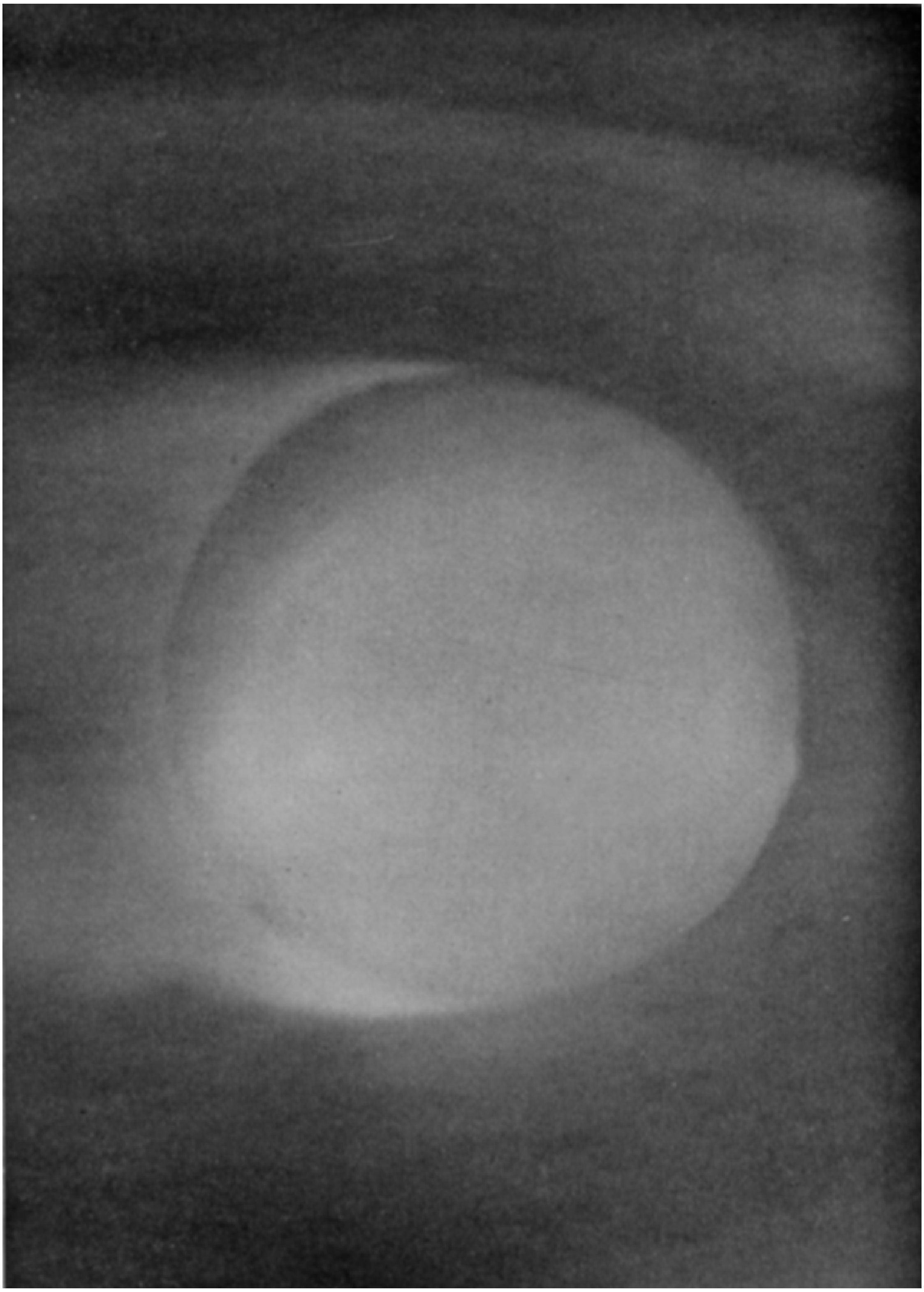


FIG. 9.

(Negative, for Positive see Frontispiece) motion but when we remember that, according to the Eulerian theory, the lines of flow should carry the smoke along a symmetrical path to the opposite pole of the sphere, as in Fig. 45 (Chap. III.), the conclusion is plain.

The author has succeeded in photographing the flow round a cylinder in motion in a smoke laden atmosphere (Fig. 9). In this example it may be noticed, that the surface or stratum of discontinuity arises from a line some distance in front of the plane of maximum section; the difference in the behaviour of a cylinder and sphere in this respect is due to the fact that in the former case the lines of flow are cramped laterally, the motion being confined to two dimensions, whereas in the latter case, the motion being in three dimensions, the fluid can "get away" with greater facility. This difference is reflected in the lower coefficient of resistance found experimentally for the sphere than that ascertained for the cylinder. Thus in the experiments of Dines (§ 226) the *pressure per square foot of maximum section* on a $^5/_8$-in. cylindrical rod was found to be more than double that on a 6-in. sphere, though doubtless the difference in size in the bodies compared may contribute something to the disparity.

The theory of discontinuity also receives support of the most convincing description from the experiments of Hutton, 1788, and Dines, 1889, by which it is shown that the pressure on a *solid hemisphere*, or a *hemispherical cup* (such as used on the Robinson anemometer), both of spherical presentation, does not differ from that on a complete sphere to an extent that experimental will disclose. This not only disposes of the streamline sphere of mathematical conception, but proves at the same time the approximate constancy of wake pressure under variation of rear body form. The same lesson is to be gleaned from experiments in the case of the *hemisphere*, *cone*, and *circular plate* (all in base presentation), whose resistance is found to be approximately equal (Fig. 17).

§ 22. Wake and Counterwake Currents.—Reference has already been made to the *frictional wake current* to which streamline body gives rise owing to the viscous stress it exerts on the fluid in its neighbourhood. With bodies of imperfect form there is, in addition to the frictional wake, a wake current constituted by the contents of the dead-water region, that is, the fluid contained within the surface of discontinuity.

The general motion of the wake current is in the same direction as the body itself, but, owing to the viscous drag exerted on it by the surrounding stream, this motion has superposed on it one of circulation, which probably results in the central portion of the wake travelling actually faster than the body[1] and the outer part slower, though Dines' experiments seem to point to the disturbance being of so complex a character that it is impossible to trace any clearly defined system.[2]

Now, since there can be no momentum communicated to the fluid in sum (§ 5), there must be surrounding the dead-water or wake current a counter-current in the opposite direction to that of the wake, that is, in the reverse direction to the motion of the body; and this counterwake current is being continuously generated, just as the wake current itself, and contains momentum equal and opposite to that of the wake. When in a fluid possessing viscosity the wake and counterwake currents intermingle by virtue of the viscous connection between them and become involved in a general turbulence, the plus and minus momenta mutually cancel, and the final condition of the fluid at all points is one of zero momentum.

We may regard the counterwake current as a survival of the motion which, we have shown, must exist in the neighbourhood of the maximum section of a streamline body (§ 13) opposite in direction to its motion through the fluid. The failure of the stream to close in behind the body means that this motion will persist.

The mingling of the wake and counterwake may be regarded as a phenomenon quite apart from the initial disturbance, and the

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[1] Since writing this passage the author has observed this "overtaking" current photographed in Fig. 9. It may be faintly discerned in this Figure in the central region of the "dead-water."

[2] On Wind Pressure upon an Inclined Surface," Proc. Royal Soc., 1890.

turbulence or otherwise of the wake does not materially add to or detract from the pressure on the front face of the body, but concerns merely the ultimate disposal of the energy left behind in the fluid.

No distinction is necessary between the frictional wake and the dead-water wake so far as the production of a counterwake current is concerned. The total wake current is the sum of the two, and the total counterwake is equal and opposite to the total wake.

§ 23. Streamline Motion in the Light of the Theory of Discontinuity—The theory of kinetic discontinuity presents the subject of streamline motion in a new light, and enables us to formulate a true definition of streamline form. Thus—

A stream line body is one that in its motion through a fluid does not give rise to a surface of discontinuity.

In the previous discussion, § 9 *et seq.*, no attempt has been made to delineate streamline form, that is to say (according to the present definition), the form of body that in its motion through a fluid will not give rise to discontinuity. It has been assumed that such a body is a possibility, and from the physical requirements of the case the general character of the body form has been taken for granted.

Under our definition, if, as in the mathematical (Eulerian) theory, we assume *continuity* as hypothesis, then all bodies must be streamline, which is the well-known consequence. If, on the other hand, as in the Newtonian medium, *we assume discontinuity*, then it is evident by our definition that streamline form can have no existence, which, again, is what we know to be the case. It remains for us to demonstrate, on the assumption of the properties of an ordinary fluid, the conditions which govern the existence or otherwise of discontinuity, and so control the form of a streamline body.

In order that streamline motion should be possible such motion must be a stable state, so that, if we suppose that by some means a surface of discontinuity be initiated, the conditions must lie such that the form of motion so produced is unstable.

Let us suppose that we have (Fig. 10) a streamline body made in two halves, and that the rear half, or run , be temporarily removed then a surface of discontinuity will be developed, as indicated in the figure. Let now the detached portion be replaced. Then the question arises, What are the changed conditions that will interfere with the permanence of the discontinuous system of flow, as depicted in the figure?

If, in the first place, the fluid be taken as inviscid, and if,

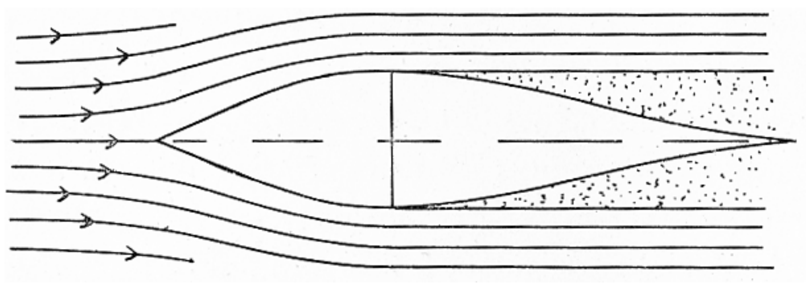

FIG. 10.

for the purpose of argument, we assume that the system of flow indicated in the figure is possible in an inviscid fluid, then it is evident that when the run is replaced we shall not have disturbed the conditions of flow, for our operations have been confined to the dead water region, where the fluid is at rest relatively to the body. Consequently the discontinuous system of flow will persist. That is to say, *under the supposed conditions* streamline motion is either unstable or is at best a condition of neutral equilibrium. Let us next introduce viscosity as a factor. The conditions are now altered, for the fluid in the dead-water region is no longer motionless, but is in active circulation, and the introduction of the rear half of the body obstructs the free path of the fluid, so that, as the outer layers of the dead-water are carried away by the viscous fluid in the interior has difficulty in finding its way back to take its place. This difficulty is greatest in the region from which the discontinuity springs, where the dead-water runs off to a "feather edge,'' and it is evident that some point of attenuation is reached at which the return flow becomes impossible, and the fluid will be "pumped out'' or ejected from the region forward of this point. This brings the discontinuity further aft on the body, where the process can be supposed repeated so that eventually the whole dead-water has been pumped away, and streamline motion supervenes. It is evident that the process will not occur in stages, as above suggested, but will be continuous.

It might be supposed from the foregoing argument that the degree of curvature of the surface of the body would not be a matter of importance, as in any case the *feather edge* of the dead-water would be sufficiently fine to ensure the ejection of some small amount of the fluid, and this process by continuous repetition would eventually clear the wake of its contents. If the surface of the body were *frictionless*, doubtless this might be the case, but it is established that there is *continuity* between the surface of an immersed body and the surrounding fluid; that is to say, there is the same degree of viscous connection between the fluid and the surface as there is between one layer of the fluid and another. The consequence of this is that the dead-water never fines off entirely, but extends forward as a sort of sheath enveloping the whole surface of the body, and if the Curvature at any point is too rapid, the ejection may not prove effective, and the discontinuity will persist. It is evident therefore that there will be some relation between the bluffness of form permissible and the viscosity of the fluid, and, other things being equal, the less the viscosity the finer will have to be the lines of the body. The theory evidently also points to the importance of *smoothness of surface* when the critical conditions are approached.

The subject is not yet exhausted. We know that the thickness of the stratum of fluid infected by skin friction increases with the distance from the "cut-water"; that is to say, the factor on which the curvature of the surface probably depends is relatively more important on the buttock than on the shoulder. Hence we may expect that the lines of entrance can with impunity be made less fine than the lines of the run.

Again, all forces due to the inertia of the fluid vary as the

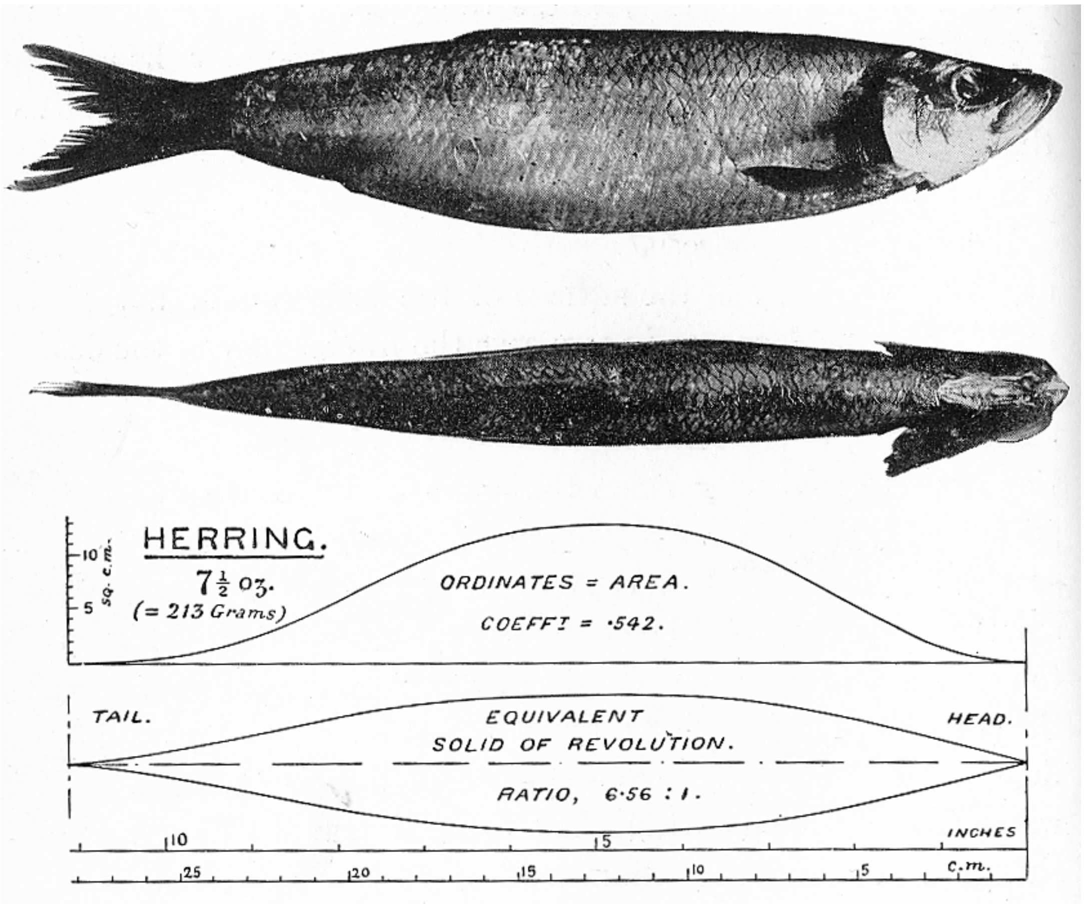

FIG. 11.

square of the velocity; those due to viscosity vary in the direct ratio of the velocity (§ 31). Therefore for different velocities the influence of viscosity predominates for low velocities, and that of inertia when the velocity is high. Consequently the form suited to high velocity will be that appropriate to low viscosity, and *vice versâ*; that is to say, the higher the velocity the finer will be the lines required.

§ 24. Streamline Form in Practice.—The practical aspect of streamline form may be best studied from the bodies of fishes and birds, the lines of which have been gradually evolved by nature to meet the requirements of least resistance for motion through a fluid, water or air, as the case may be.

Since all animals have functions to perform other than mere

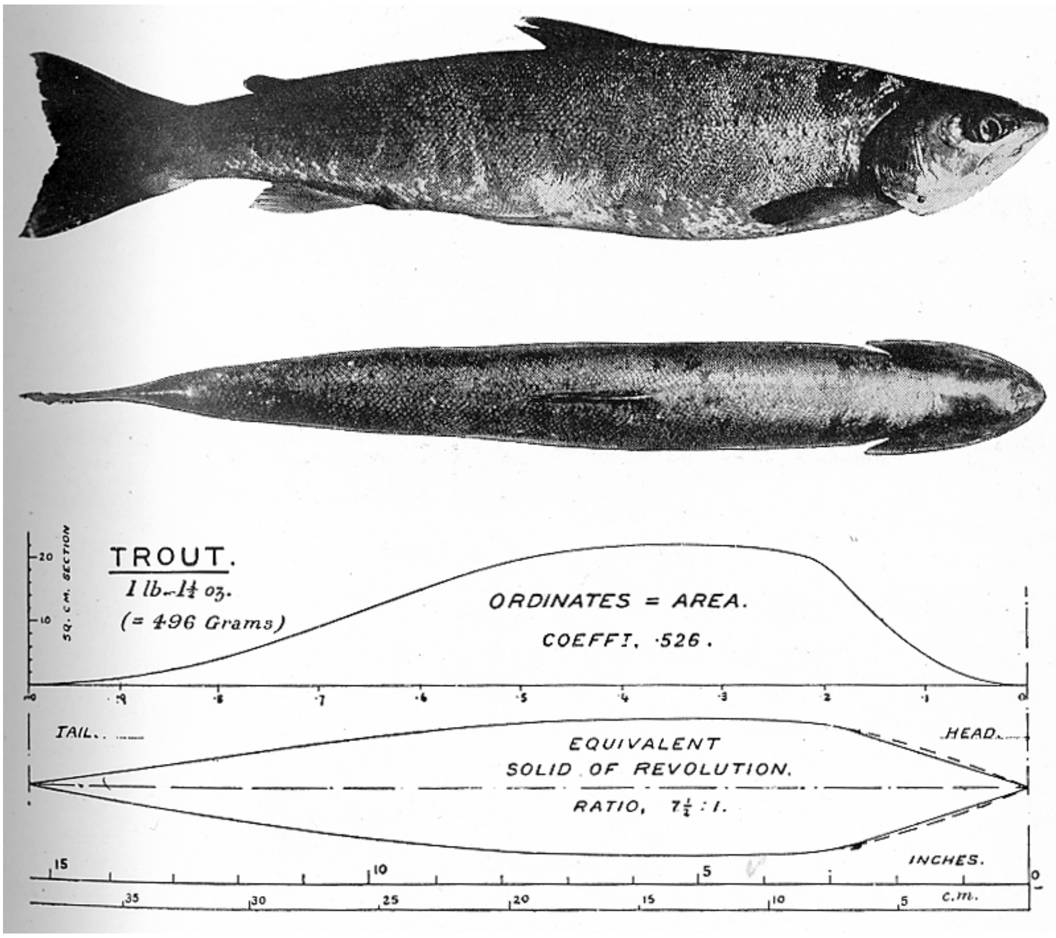

FIG. 12.

locomotion, we find great diversity of detail, and we frequently meet with features whose existence is in no way connected with the present subject. We may readily recognise in these cases the exceptional development of certain organs or parts to meet the special requirements of a particular species, and by a sufficiently wide selection we can eliminate features that are not common, and so arrive at an appreciation of that which is essential. Thus the *herring* (Fig. 11), the *trout* (Fig. 12), or

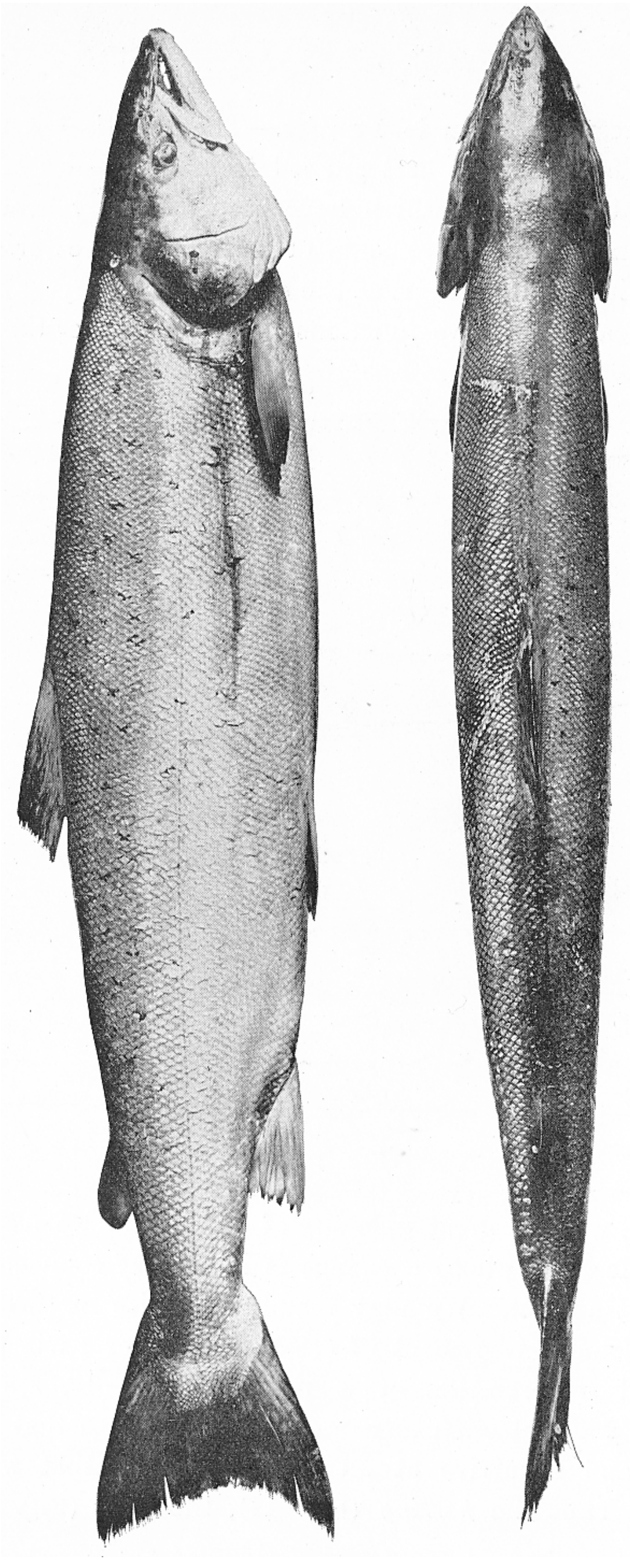

FIG. 13.

the *salmon* (Fig. 18) may be cited as typically *fish-shaped fish*.

Beyond the lessons to be derived from these natural forms, there is very little practical information available. The lines of ships are governed by considerations foreign to the subject, the question of wave-making, for example, being a matter of vital importance. The *submarine* has not yet reached a stage of development that would justify its form being taken as a fully

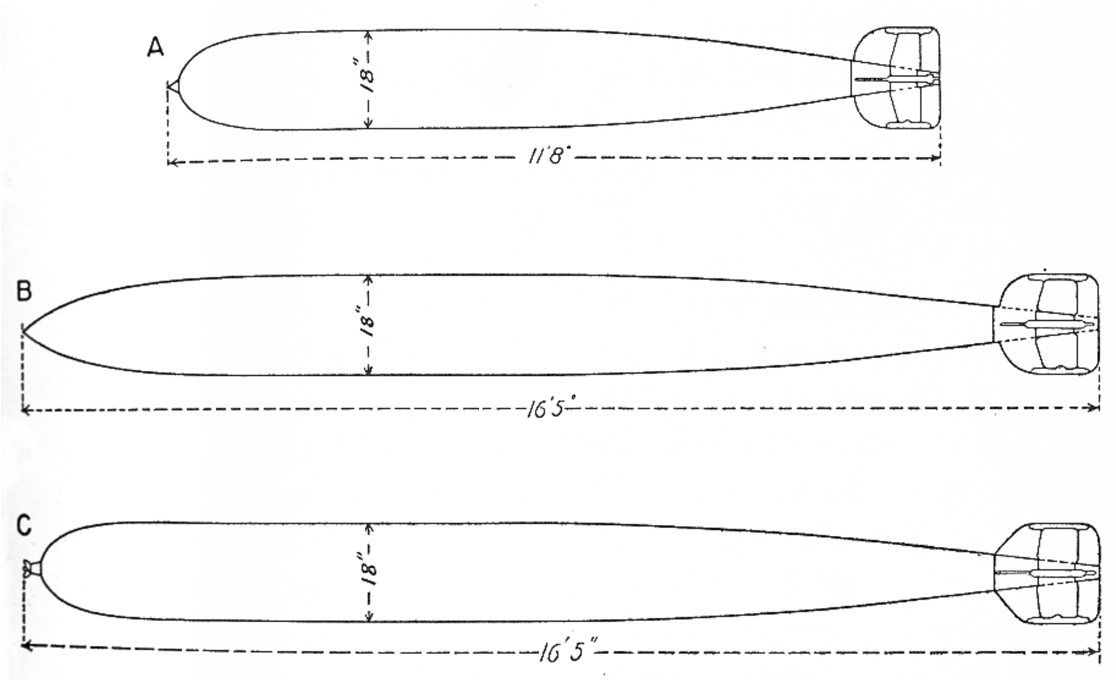

FIG 14.

evolved model: also, for obvious reasons, this type of vessel is one of which but little information has been published.

In Figs. 11 and 12 curves are given whose ordinates represent the area of cross-section at different points. This curve has been obtained by differentiating a displacement curve plotted from a Series of immersion measurements. These measurements were made by a method of displacement, the fish, suspended tail downward, being lowered stage by stage into a vessel of water, measurements being made of the overflow.

The area curves have been further translated into the form of *solids of revolution*, which may be taken as the equivalent of the original form in each case. Some doubt exists as to the exact form in the region of the head, owing to the water entering the gills. The effect of this is very evident in the case of the trout (Fig. 12), where the form has been "made good" by a dotted line.

For the purpose of comparison outline elevations are given in Fig. 14 of three types of Whitehead torpedo. These are forms that have been developed by long experience, but the shape is largely dictated by special considerations. The bluff form of head, for example, in models A and C is adopted in order to bring the' explosive charge into as close proximity as possible to the object attacked. It probably also gives a form that is more easily steered.

§ 25. Streamline Form.—Theory and Practice Compared.—Before a rigid comparison can be instituted between the theoretical results of § 23 and the actual forms found in nature considerable further information is required. We do not know with accuracy the speeds for which the different fish forms have been designed or are best adapted. We also lack knowledge on certain other important points. The present comparison must therefore be confined to generalities.

In the first place, we may take it that the conclusion as to the. bluffer form being that suited to greater viscosity is fully borne out in practice, though the whole of the considerations bearing on this point are not here available. It is explained in Chap. II. that the viscosity *divided by density* (or *kinematic viscosity*) is the proper criterion in such a case as that under discussion, and on this basis air is far more viscous than water, so that we shall expect to find aerial forms bluffer in their lines than aqueous forms. Taking the solid of revolution as the basis of comparison, we have in the case of the herring and the trout the length approximately seven times the maximum diameter. The general ratio found amongst bird forms is about three or four to one, the samples chosen for measurement being as far apart as the albatros and the common sparrow.

Consequently we find that the theoretical conclusion receives substantial confirmation.

The relation of fineness to speed is not so easy of demonstration, owing to the absence of accurate data. It would, however, seem to be sufficiently obvious as a matter of general experience that our conclusions hold good. It is almost certain that in general the fish with the finer lines are the faster swimmers. If this conclusion be accepted, the viscosity relation of the preceding paragraph is emphasised, for there is no doubt that the average speed of flight is greatly in excess of any ordinary velocity attained by fish.

§ 26. Mutilation of the Streamline Form.—There are certain types of body that may be regarded as mutilations of the streamline form, and the consequences of such mutilation may now be examined.

If, in the case of a body propelled at a constant velocity, the entire run be removed, as in § 23, the consequence is a surface of discontinuity emanating from the periphery of section. Under these circumstances, if we neglect the influence of viscosity and the consequent loss of wake pressure, the work done appears wholly in the counterwake current, on the production of which energy is being continuously expended. This performance of work is otherwise represented by a resistance to motion, being the difference between the excess pressure on the head and the diminished pressure on the shoulder, according to the principle explained in § 11. If now we restore the buttock, so that the mutilation is confined to the simple loss of the tail (Fig. 15), the diminished pressure on the buttock acts as a drag upon the body, and more work must be expended in propulsion. This additional energy will appear in the fluid as a radial component in the motion of the stream which does not exist if the whole run is removed. It is probable that some of this energy is restored by an increase in the pressure of the dead water due to the converging stream, but we have no means of making a quantitative computation.

An illustration of this principle may be cited in the type of hull employed in a modern racing launch. The stern is cute. square and clean, and may constitute the maximum immersed section. There would seem in fact to be no logical compromise between a boat with an ordinary well-proportioned entrance and run, and one in which the latter is sacrificed entirely. In such

form, when travelling at high speed the water quits the transom entirely, and consequently sacrifice is made of the hydrostatic pressure on the immersed transom area. The point at which

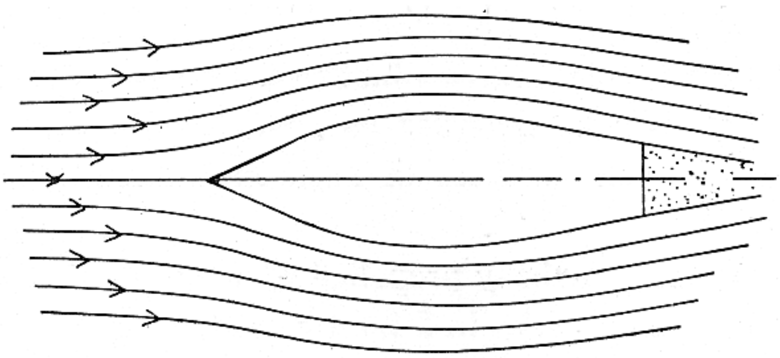

FIG. 15.

the *front half* of a boat thus takes less power for its propulsion than the whole is probably about that speed at which the skin friction on the *run* (the after-half), if present, exceeds the hydrostatic pressure on the maximum immersed section. This does not, however, determine the point at which it pays to make the sacrifice, owing to the fact that for the same capacity the truncated form has to be that of a larger model. The rating rule also exerts an arbitrary influence. When, as is usual, the length is penalised, an additional inducement is offered for the designer to adopt the truncated type. When the truncated type of hull is adopted it is advantageous to employ shallow draught, for the hydrostatic pressure for a given displacement is less. This form is also partly dictated by considerations relating to propulsion.

§ 27. Mutilation of the Streamline Form (continued).—In Fig. 16, A and B, the consequences of truncating the fore body, or *entrance*, of a streamline body are indicated diagrammatically. If, as in A, the mutilation be slight, the result may be merely a local disturbance of the lines of flow. A surface of discontinuity will probably arise, originating and terminating on the surface of the body in the manner shown. It is possible that if the streamline body be travelling at something approaching its critical velocity (at which even in its complete form it is on the point of giving

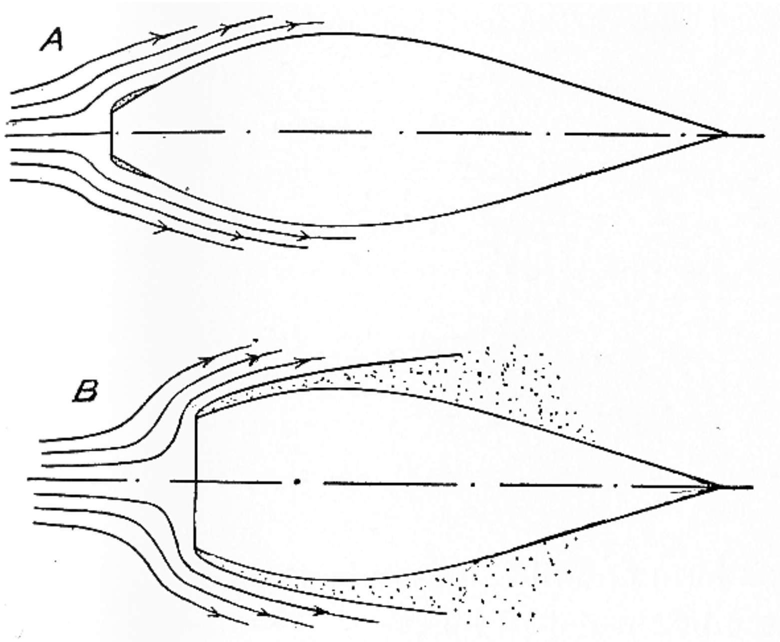

FIG. 16

rise to discontinuity), a minor mutilation such as here suggested might have more serious consequences.

If the greater part of the entrance be removed, as shown at B, the surface of discontinuity generated quits the body for good, and the resistance becomes immediately as great as that of a normal plane of area and form equal to that of the section. This is in harmony with the experiments of Hutton and Dines, to which reference has already been made (Fig. 17), the three bodies shown being found to offer the same resistance within the limits of experimental error.

It is evident that the dictum of the late Mr. Froude, that it is "blunt tails rather than blunt noses that cause eddies" (and therefore involve a loss of power), is applicable only to bodies having already some approximation to streamline form. It is obviously useless to provide a nice sharp tail if previous attention has not been given to the shoulder and buttock lines. Mr. Froude probably meant that in a well-designed streamline form the tail should be finer in form than the head, a matter that up to his time had presumably been neglected.

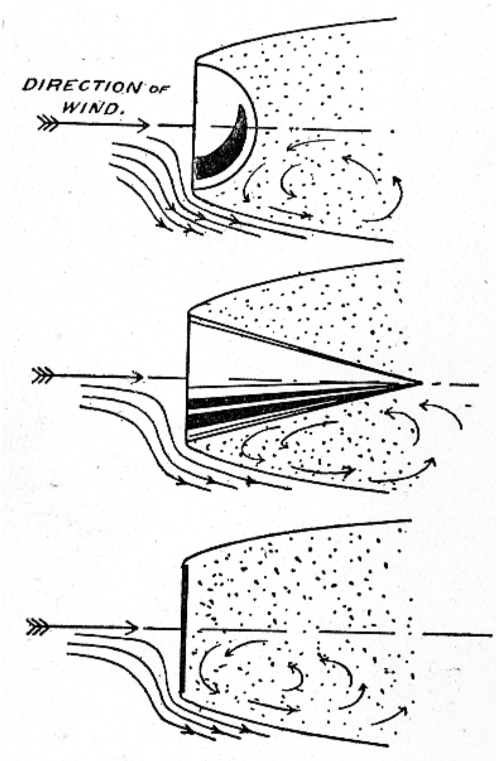

FIG. 17.

The primary importance of easy shoulder lines has been long recognised as a fundamental feature in the design of projectiles. A full - sized - section of a Metford .303 bullet, illustrating this point, is given in Fig. 18, and a streamline form of which it may be regarded as a "mutilation" is indicated by the dotted line.

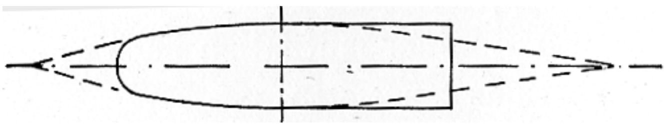

FIG. 18.

§ 28. **Streamline Flow General.**—Let us suppose an approximate streamline form to be built of bricks, and, in the first place, we will assume that the bricks are so small as to merely give rise to a superficial roughness. Then this roughness will add to the skin friction and will give rise to some local turbulence, but the general character of the flow system remains as before. We may go further and suppose the bricks so large as to form steps capable of giving rise to surfaces of discontinuity (Fig. 19). Then the resistance will be increased, and the layer of fluid next the body will be violently stirred up; but if we examine the fluid some distance away we shall still find it comparatively unaffected. If we now suppose the body to consist of a few large blocks, the depth of fluid affected by turbulence will be greater, but at a sufficient distance away we may still expect to find lines of flow of characteristic streamline form. We may therefore generalise and say, *All bodies passing through a fluid are surrounded by a streamline system of flow of a greater or less degree of perfection depending upon the conformability or otherwise of the face or surfaces of the body.*

This proposition, if not sufficiently obvious from the

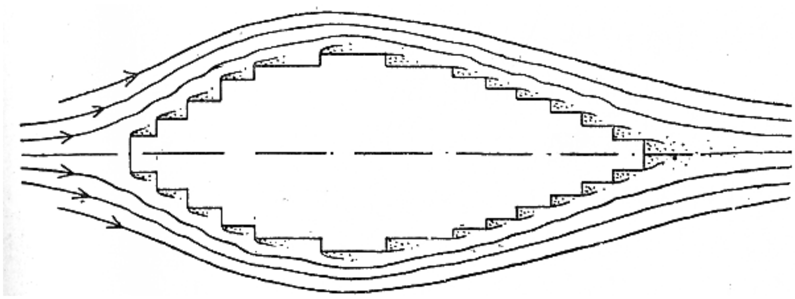

FIG. 19.

considerations above given, may easily be demonstrated experimentally.

In the experiment described in § 17, the orbital motion of the particles of the fluid is demonstrated by the motion of an ichthyoid body in air irregularly charged with smoke. This orbital motion, with its consequent displacement, is quite characteristic, and if other shapes of body be substituted for the streamline form, the motion of the fluid a short distance away is not sensibly affected In the case of a body of streamline form, the motion can be observed much closer to the axis of flight than is the case for a sphere or other bluff form; also when the movement is complete nothing further happens. In the case of a sphere, the looked-for movement duly takes place; built immediately after the whole of the fluid under observation is involved in a state of seething turbulence, where the wake and counterwake currents are mingling. It the point of observation is sufficiently remote, the orbital motion may be detected, even in the case of the normal plane, beyond the immediate reach of the wake turbulence.

§ 29. Displacement due to Fluid in Motion.—It has been shown (§ 11.5) that the fluid in the neighbourhood of the path of flight of a streamline body undergoes displacement, and that the total displacement is equal to the volume of the body. It might be expected in the case of the normal plane, which possesses no volume, that the displacement would be nil, and such would doubtless be the ease if the term of flow were that of the Eulerian theory.

In actuality the normal plane, in common with bodies of bluff form, carries a quantity of fluid bodily in its wake, which from the present point of view becomes in effect part of the body, so that the displacement manifests itself just as if the plane were possessed of volume. This is characteristic of all bodies that give rise to discontinuous motion; the displacement is greater than the actual volume of the body. If there were no mingling of the wake and counterwake currents, the displacement would be infinite, for the counterwake current would persist indefinitely.

In the case of a streamline body, a certain amount of fluid is carried along with the body by viscosity, and this similarly increases the effective displacement volume.

It would appear from actual observation that, where the displacement is due to the attendant fluid, the outer streamlines have a motion closely resembling that produced by a streamline body, but that those nearer the axis of flight terminate in the turbulent wake; the commencement of the orbit is all that can be seen.

§ 30. Examples illustrating Effects of Discontinuous Motion.—On the practical importance of the study of motion of the discontinuous type it is unnecessary to dwell. It is at present the only basis on which it is possible to account for the phenomenon of fluid resistance as experimetally known. Beyond this there are many examples and illustrations which are of especial interest, considered either as proofs of the theory itself or in relation to their actual consequence or utility.

A useful application of the principle is found in the screen employed fast steamships to protect the navigating officer, and frequently the "watch," from the rush of air, without

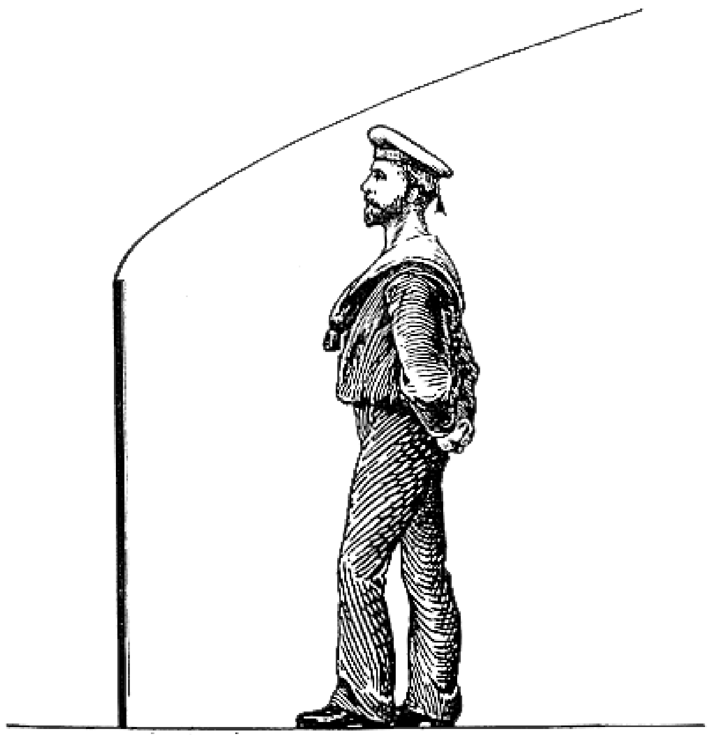

FIG. 20.

obstructing the field of vision. This illustrated diagrammatically in Fig. 20, in which it will be seen that the live Stream is carried clear over the sailor's head, the latter being protected by the surface of discontinuity. A similar device is frequently adopted in connection with the dashboard of a motor car.

Evidence of the most striking kind of the existence of a surface of discontinuity is sometimes met with in the growth of trees in the immediate vicinity be seen that of the edge of a cliff (Fig. 21). It may the form of the surface is clearly delineated, the tree top being cut away as though it might have been sheared off by a stroke of a mighty scythe.

An interesting example of an indirect effect of discontinuity is to be found in the effect of "cut" or "side" on the flight of ball. Let a ball (Fig. 22) moving in the direction of the arrow *A* have spin in the direction of the arrow *B*. Now where the

FIG. 21.

direction of motion of the surface of the ball is the same as the relative motion of the fluid, as at D, the surface will assist the stream in ejecting the dead water, so that the discontinuity will be de1ayed, and will only make its appearance at a point some distance further aft than usual. On the other hand, on the side that is opposing the stream the surface of the tall will pump air in, and so assist the discontinuity, which will make its appearance prematurely. The net result of this is that the counterwake will have a lateral component (downwards in the figure), and, on the principle of the continuous communication of momentum there will be a reaction on the ball in the opposite direction, that is to say upwards. A ball may therefore be sustained gravity or be made to "soar" by receiving a spin in the direction shown, or, if the spin be about a vertical axis, the path of the ball will be a curve (in plan), such that the aerodynamic reaction will be balanced by centrifugal force.

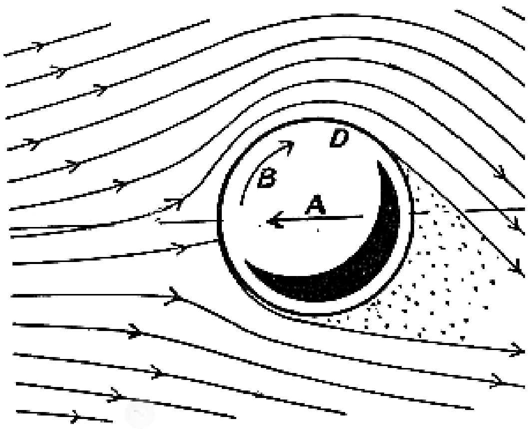

FIG. 22.

The actual means by which the reaction acting on the ball comes about may be understood from either of two points of view. We may (Fig. 22) regard this reaction as the centrifugal effect of the air passing over the ball preponderating greatly over that of the fluid passing underneath, or if we anticipate a knowledge of hydrodynamic theory (Chap. III.), we know that the greater proximity of the lines of flow in the former region is alone sufficient to indicate diminished pressure. The lines as drawn in the figure are not plottings—there is no known of plotting a field of flow of this degree of complexity —but they may be taken as a very fair representation of what the plotting would be if it could be effected.

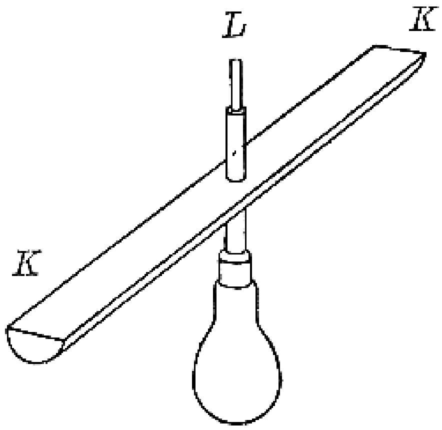

FIG. 23.

The reason that the streamlines have been shown rising to meet the ball in its progress will be better understood in the light of

Chaps. III. and IV. This detail is related to move advanced considerations than can he entered into at present.

A further interesting example is found in the *aerial turbillion*[1] (Fig. 23), in which the rotor K is a stick of segmental section mounted to revolve freely about the axis L, The plane face of the rotor is set truly at right angles to the axis of rotation. If this apparatus be held in a current of air with the plane face fronting the wind, as, for instance, by holding it outside the window of a railway carriage in motion, the rotor evinces no tendency to go round in the one direction or the other. If, however, a *considerable* initial spin be imparted in either direction, the wind will suddenly *get a bite*, so to speak, and the rotor will gather speed

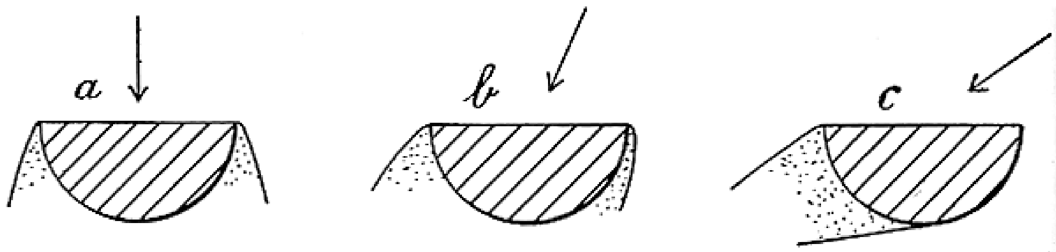

FIG. 24.

and spin at an enormous rate, as if it were furnished with sails like a well-designed windmill.

Referring to Fig. 24, we have at a the type of flow illustrated to which the blade of the rotor will give rise when its motion is normal to the air; *b* similarly indicates the form of flow when the rotor is going round slowly, not fast enough for the air to take hold. In both these figures we have the flow independent of the "rear body form," and the rotor behaves just as if it were a flat plate. Now, let us suppose that the rotor be given sufficient initial spin to bring about the state of things represented at *c*.

[1] This interesting aerodynamic puzzle was first brought to the notice of the author by Mr. Henry Lea, consulting engineer, of Birmingham, who, it would appear, had it communicated to him by Mr. A. S. Dixon, who in turn had it shown him when travelling in Italy by Mr Patrick Alexander. The author has taken no steps to trace the matter further. The explanation here given is his own.

The surface of discontinuity that ordinarily springs from the leading edge has got so close to the rear body of the rotor as to have *ejected the "dead -water"* on that side, and the resulting form of flow will he something like that illustrated in Fig. 25. Here the pressure on the left-hand side (as shown) will be that of the "dead-water," which is, as we know, somewhat less than that of hydrostatic head, while that on the right-hand side will, owing to the centrifugal component the stream, be very much lower; that as to say, the rotor will experience a force acting from left to right which is in the direction of the initial spin, so that the motion will be accelerated and will continue. The fact that the propelling force only comes into existence when the initial spin is sufficient to eject the dead water from the leading side of the rotor blade fully explains the observed fact that a very considerable initial spin is necessary.

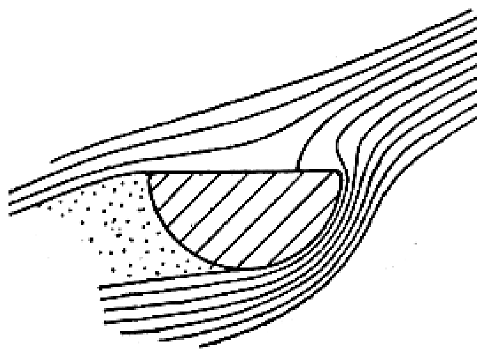

FIG. 25.